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(0,2) Noncritical Strings in Six Dimensions

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Abstract

Type IIB strings compactified on K3 have a rich structure of solitonic strings, transforming under $SO(21, 5, \mathbb{Z})$. We derive the BPS tension formula for these strings, and discuss their properties, in particular, the points in the moduli space where certain strings become tensionless. By examining these tensionless string limits, we shed some further light on the conjectured dual M-Theory description of this compactification.

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1. Introduction

In the past two years we have witnessed a revolution in our understanding of non-perturbative physics. By looking closely at singular points in the moduli space of solutions to string vacua new fields massless degrees of freedom were discovered, which describe the local dynamics of the singularity. Some examples are Strominger's resolution of the conifold [1], and Witten's resolution of small $SO(32)$ instantons [2]. In both of these cases the new massless states at the singularity are particles which are either hypermultiplets - massless solitonic states, and/or vector multiplets - enhanced gauge symmetries.

On the other hand, the resolution of some singularities has involved the introduction of new light degrees of freedom which are *not* particles. Witten's resolution of the singularities of type IIB compactified on K3 [3] involved the introduction of tensionless strings. This is rather exotic infrared physics, of a sort that had not been seen hitherto. Subsequently, tensionless strings have been found to be responsible for nontrivial infrared physics in a variety of string theory contexts [4,5]. The discovery of new nontrivial infrared physics is always exciting, especially when it seems to be exotic from the field theory perspective. It is very compelling to try to understand these tensionless string theories better.

In this note, we study the bound state spectrum of the BPS saturated strings which arise in the original context of type IIB compactified on K3. Upon further compactification to 5 dimensions, this theory becomes dual to the heterotic string on T^5 , and we can exploit the known perturbative behaviour of the heterotic string to learn some nonperturbative features of this theory.

2. $(0, 2)$ Supersymmetry in 6 Dimensions

As the vector of $SO(5, 1)$ appears in the antisymmetric product of two 4s, spinors in 6 dimensions can be taken to be symplectic-Majorana-Weyl. So the supercharges, Q_α^a of chiral $(0, N)$ supersymmetry carry both a spinor index, α , and an $Sp(N)_R$ index, a . For $N = 2$, the case of interest in this note, the supersymmetry algebra is

$$\{Q_\alpha^a, Q_\beta^b\} = 2\omega^{ab}\gamma_{\alpha\beta}^\mu P_\mu + \gamma_{\alpha\beta}^\mu Z_\mu^{ab} \quad (2.1)$$

where ω^{ab} is the $Sp(2)$ -invariant tensor. Z_μ^{ab} is a central charge of the supersymmetry algebra which transforms as a Lorentz vector and as a **5** of $Sp(2)_R$.

As the central charge is a vector, the associated gauge field is a 2-form, which naturally couples to a *string*, rather than a *particle*.

The massless representations of (2.1) (the central charge vanishes for particles) are

$$\begin{aligned} \text{gravity} & \quad (3, 3; \mathbf{1}) \oplus (3, 2; \mathbf{4}) \oplus (3, 1; \mathbf{5}) \\ \text{tensor} & \quad (1, 3; \mathbf{1}) \oplus (1, 2; \mathbf{4}) \oplus (1, 1; \mathbf{5}) \end{aligned} \tag{2.2}$$

where we denoted the representations of the little group, $Spin(4) (= SU(2) \times SU(2))$ and $Sp(2)_R$.

A 2-form has a 3-form field-strength which, in 6 dimensions, can be broken into a self-dual and an anti-self-dual piece. In light-cone gauge, these correspond to self-dual 2-forms, $(3, 1)$ of $Spin(4)$, and anti-self-dual 2-forms, $(1, 3)$ of $Spin(4)$. The former are part of the gravity multiplet; the latter are part of tensor multiplets. The central charge in (2.1) measures the strength of the coupling of a string to the 5 self-dual 2-forms in the gravity multiplet (which transform as a $\mathbf{5}$ of $Sp(2)_R$).

Cancellation of gravitational anomalies require that the number of tensor multiplets be equal to 21 [6,7]. Each tensor multiplet contains 5 real scalars, transforming as the $\mathbf{5}$ of $Sp(2)_R$. Consistent coupling to supergravity requires [8] that these 105 real scalars parametrize a manifold which is, at least locally, of the form

$$\frac{SO(21, 5)}{SO(21) \times SO(5)} \quad .$$

The massless bosonic fields of the theory, thus, consist of the graviton, 5 self-dual 2-forms, 21 anti-self-dual 2-forms and these 105 scalars. This is precisely what emerges as the low-energy limit of Type II-B strings compactified on $K3$.

The 2-forms arise as follows. The NS-NS 2-form, B , and the R-R 2-form, \tilde{B} , whose indices lie in the 6 noncompact directions each break up into a self-dual and an anti-self-dual piece. The 2^{nd} homology group, $H_2(K3)$ has signature $(19, 3)$. Integrating the self-dual R-R 4-form over a (anti-)self-dual 2-cycle on $K3$ yields an (anti-)self-dual 2-form in 6 dimensions. All in all, we have the expected 26 2-forms:

$$\begin{aligned} B^+, \tilde{B}^+, \int_{\gamma \in H_2^+(K3)} G & \quad 5 \text{ self-dual 2-forms in the gravity multiplet,} \\ B^-, \tilde{B}^-, \int_{\gamma \in H_2^-(K3)} G & \quad 21 \text{ anti-self-dual 2-forms in the 21 tensor multiplets} \quad . \end{aligned} \tag{2.3}$$

Since there is a 26-dimensional space of 2-forms (21 anti-self-dual and 5 self-dual) to which a string might couple, we expect a rich spectrum of strings in 6 dimensions. Our aim is to find a BPS tension formula for these strings.

It is, perhaps, best to start by recalling the situation in 10 dimensions. There there is a two-dimensional space of strings, which couple to some linear combination of the two 2-forms, B, \tilde{B} . The scalars take values in the fundamental domain on the upper half-plane,

$$\tau = \tilde{\varphi} + ie^{-\varphi} \in SL(2, \mathbb{Z}) \setminus \frac{SL(2, \mathbb{R})}{U(1)} \quad (2.4)$$

where φ is the dilaton and $\tilde{\varphi}$ is the R-R scalar. The tension of a string which couples to $n_1 B + n_2 \tilde{B}$ is given by [9]

$$T^2 = \frac{1}{16\pi^2} (n_1 \ n_2) \frac{1}{\text{Im } \tau} \begin{pmatrix} 1 & -\text{Re } \tau \\ -\text{Re } \tau & |\tau|^2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} (M_{pl}^{(10)})^4 \quad (2.5)$$

This is the natural $SL(2, \mathbb{Z})$ -invariant expression, where $SL(2, \mathbb{Z})$ takes

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}, \quad \begin{pmatrix} B \\ \tilde{B} \end{pmatrix} \rightarrow \begin{pmatrix} dB - b\tilde{B} \\ -cB + a\tilde{B} \end{pmatrix}, \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \rightarrow \begin{pmatrix} an_1 + bn_2 \\ cn_1 + dn_2 \end{pmatrix} \quad (2.6)$$

In addition to this set of strings which couple to B and \tilde{B} , new strings arise in 6 dimensions from wrapping the Type IIB 3-brane around 2-cycles on the $K3$. Since the 3-brane couples to the self-dual 4-form, G , these strings couple to the 2-forms we obtained by integrating G over the corresponding 2-cycle on $K3$. Once we turn on background fields, these, apparently distinct, sets of strings mix in a complicated way. Our task is to write down the analogue of (2.5).

The results are as follows. The space of scalars [10]

$$\mathcal{M} = \Gamma \setminus \frac{SO(21, 5)}{SO(21) \times SO(5)} \quad (2.7)$$

is the moduli space of *even, self-dual Lorentzian lattices of signature* $(21, 5)$ (21 negative and 5 positive eigenvalues). The discrete group Γ , usually written as $SO(21, 5, \mathbb{Z})$, is the discrete subgroup of $SO(21, 5)$ which acts as automorphisms of some chosen basepoint lattice, Λ_0 . Let $\{e_i\}$ be a basis for Λ_0 . Then, if $G(\phi)$ is an $SO(21, 5)$ transformation which takes us from the basepoint to the point $\phi \in \mathcal{M}$, $\mathbf{E}_i(\phi) = G(\phi)e_i$ is a basis for the corresponding lattice, Λ_ϕ .

The Dirac quantization condition on the strings simply states that the allowed charges, under the $(21, 5)$ -dimensional space of 2-forms, lie on this even, self-dual Lorentzian lattice. That is, an allowed charge vector is an integer linear combination $\sum_i n_i \mathbf{e}_i$. Of course, the description in terms of a fixed basis of 2-forms is good only locally in the moduli space. Under

the action of the modular group, Γ , the 2-forms transform as the fundamental 26-dimensional representation. So it is more convenient for our purposes to work with the “charge vector” $\mathbf{q}(\phi) = \sum_i n_i \mathbf{E}_i(\phi)$, a section of a flat vector bundle over \mathcal{M} with structure group Γ .

The condition for a string to be BPS-saturated requires first that

$$\mathbf{q}^2 \geq -2 \quad . \quad (2.8)$$

Introduce the orthogonal projection, P , which projects any vector onto the 5-dimensional positive-signature subspace. Define

$$\mathbf{q}_+ = P\mathbf{q}, \quad \mathbf{q}_- = (\mathbb{1} - P)\mathbf{q} \quad . \quad (2.9)$$

In particular, \mathbf{q}_+ simply measures the strength of the coupling of the string to the 5 self-dual 2-forms in the gravity multiplet. The BPS tension formula is

$$T^2 = \frac{1}{8\pi^2} |\mathbf{q}_+(\phi)|^2 (M_{pl}^{(6)})^4 \quad (2.10)$$

where, here, $M_{pl}^{(6)}$ is the *6-dimensional* Planck mass.

Consider a BPS-saturated string with charge-vector $\mathbf{q} = \mathbf{q}_1 + \mathbf{q}_2$ equal to the sum of the charge-vector of two other BPS-saturated strings. This string is stable against decaying into the two other strings provided

$$T < T_1 + T_2 \quad . \quad (2.11)$$

By the triangle-inequality and (2.10), we always have

$$T \leq T_1 + T_2$$

with equality only for $(\mathbf{q}_1)_+ = c(\mathbf{q}_2)_+$ for some nonnegative constant c .

2.1. The fundamental string

Where is the fundamental Type IIB string? Consider a subspace of \mathcal{M} on which there is a distinguished even self-dual sublattice of dimension $(1, 1)$, corresponding to the anti-self-dual and self-dual components of B . There are two strings corresponding to the basis vectors of this lattice. The first, corresponding to the basis vector $\mathbf{e}_1 = \frac{1}{\sqrt{2}}(1, 1)$, couples to the sum $(B^- + B^+) = B$, and is the fundamental Type IIB string. The second, corresponding to the basis vector $\mathbf{e}_2 = \frac{1}{\sqrt{2}}(1, -1)$, couples to the difference $(B^- - B^+)$ and is solitonic. There is a 1-parameter family of such distinguished (even, but not self-dual) sublattices, corresponding to

$$\begin{aligned} \mathbf{E}_1 &= e^\varphi \mathbf{e}_1 \\ \mathbf{E}_2 &= e^{-\varphi} \mathbf{e}_2 \quad . \end{aligned} \quad (2.12)$$

The tensions of these strings are (2.10)

$$T_1^2 = \frac{1}{16\pi^2} e^{2\varphi} (M_{pl}^{(6)})^4, \quad T_2^2 = \frac{1}{16\pi^2} e^{-2\varphi} (M_{pl}^{(6)})^4 \quad . \quad (2.13)$$

Recalling that, in d dimensions,

$$M_{pl}^{(d)} = e^{-\frac{2}{d-2}\varphi} M_s \quad (2.14)$$

we see that, as expected, the former has a tension equal to $\frac{M_s^2}{4\pi} = \frac{1}{2\pi\alpha'}$, whereas the latter is very heavy at weak coupling ($T_2 = \frac{e^{-2\varphi}}{2\pi\alpha'}$).

The string with charge vector

$$\mathbf{q} = \mathbf{E}_1 + \mathbf{E}_2 = \sqrt{2}(\cosh \varphi, \sinh \varphi)$$

couple to the anti-self-dual part of B . At $\varphi = 0$, the tension of this string

$$T = \frac{1}{2\pi} (M_{pl}^{(6)})^2 |\sinh \varphi|$$

vanishes. At this point the fundamental string and the string which couples to $(B^- - B^+)$ become degenerate.

2.2. Tensionless strings

Obviously, if $\mathbf{q}_+ = 0$, then the tension of the corresponding string vanishes. Of course, \mathbf{q} cannot be identically 0; the string must have some charge. The condition (2.8) $q^2 \geq -2$ then implies that such a string has

$$q^2 = -2, \quad \mathbf{q}_+ = 0. \quad (2.15)$$

The set of such vectors spans an even (negative-definite) Euclidean sublattice of Λ_ϕ . In fact, such a set of vectors forms the root system for a simply-laced (since all the vectors have $(\text{length})^2 = -2$) Lie algebra.

When do such tensionless strings arise? Some are associated with singularities of the $K3$ surface; when certain 2-cycles on the $K3$ shrink, the corresponding wrapped 3-branes become tensionless. Such singularities naturally have an ADE classification, and we get tensionless strings associated to the corresponding ADE root system.

Similarly, as we have seen, the string which couples to the anti-self-dual part of B becomes tensionless at $\varphi = 0$ (and suitable values for the other background fields, in particular, $\tilde{\varphi} = 0$).

3. Relation to the Heterotic String in 5 Dimensions

We compactify from 6 down to 5 dimensions on a circle of radius r (in the Einstein metric). A string which wraps n times around the circle corresponds to a particle of mass

$$m = 2\pi|n|rT \quad . \quad (3.1)$$

We use T-duality to relate this to the Type IIA string compactified on $K3 \times S^1$ and then use string-string duality to relate this to the heterotic string compactified on T^5 .

Here we recognize

$$\mathcal{M} = \Gamma \backslash \frac{SO(21, 5)}{SO(21) \times SO(5)}$$

as the Narain moduli space [11,12] of heterotic strings compactified on T^5 . What we called the charge vector \mathbf{q} previously is simply the internal momentum of the toroidally-compactified heterotic string.

$$\mathbf{q}_R = \mathbf{q}_+, \quad \mathbf{q}_L = \mathbf{q}_- \quad . \quad (3.2)$$

The heterotic string coupling is given in terms of the radius of the circle by

$$e^{2\varphi_H/3} = \frac{1}{\sqrt{2}} M_{pl}^{(5)} r \quad . \quad (3.3)$$

The (perturbative) mass formula for a heterotic string state with internal momentum $\mathbf{q} = (\mathbf{q}_L, \mathbf{q}_R)$ is (in units of the heterotic string scale)

$$\begin{aligned} m^2 &= q_L^2 + 2(N_L - 1) \\ &= q_R^2 + 2N_R \quad . \end{aligned} \quad (3.4)$$

A BPS-saturated state (the only thing we have any right to compare with the Type II string) has $N_R = 0$. So, for BPS-saturated states, $m^2 = q_R^2$ and

$$q_R^2 - q_L^2 = 2N_L - 2 \geq -2 \quad . \quad (3.5)$$

Putting (3.1), (3.3) and (3.4) together, and using the relation

$$(M_{pl}^{(5)})^3 = r(M_{pl}^{(6)})^4$$

we determine the tension of the putative wrapped strings.

Of course, in outline, the proof of the tension formula (2.10) does not really rely at all on the details of the chain of dualities we have used. Once we realize that, upon compactification

on a circle of radius r , a wrapped string corresponds to a particle of mass $M = 2\pi rT$, we can obtain the BPS bound on the masses of these particles simply by manipulating the dimensionally-reduced supersymmetry algebra (2.1). The little group for this situation is $Spin(4) = SU(2) \times SU(2)$. The supercharges of (2.1) transform as doublets of one of the $SU(2)$ s and can be written as $Q_i^a, Q_a^{\dagger i}$, where i is an $SU(2)$ index. The supersymmetry algebra becomes

$$\begin{aligned}\{Q_i^a, Q_b^{\dagger j}\} &= 2\delta_b^a \delta_i^j M \\ \{Q_i^a, Q_j^b\} &= \epsilon_{ij} z^{ab} \\ \{Q_a^{\dagger i}, Q_b^{\dagger j}\} &= \epsilon^{ij} z_{ab}\end{aligned}\tag{3.6}$$

where $z^{ab} = \int_{S^1} Z_\mu^{ab} dx^\mu$ and we raise and lower $Sp(2)$ indices using ω_{ab} . The standard trick is to consider now the operator

$$A_i^a = Q_i^a - \frac{1}{2M} \epsilon_{ij} z^{ab} Q_b^{\dagger j} \quad .$$

We then have the positivity condition

$$\begin{aligned}0 &\leq \{A_i^a, A_a^{\dagger i}\} \\ &= \delta_i^j \left[8M - \frac{1}{2M} z_{ab} z^{ab} \right]\end{aligned}\tag{3.7}$$

from which we conclude $M^2 \geq \frac{1}{16} |z|^2$.

On the other hand, the connection with the heterotic string gives us some physical information which we can now exploit to argue for the existence and uniqueness of bound states of strings for a given charge vector $\mathbf{q} = \sum n_i \mathbf{E}_i$. The corresponding string is stable *if and only if* the $\{n_i\}$ have no common factor.

This follows from comparing the spectrum of wrapped strings to the perturbative spectrum of BPS states of the heterotic string on T^5 . Given a string with charge vector \mathbf{q} , if there were a stable string with charge vector $k\mathbf{q}$, then there would be, upon compactifying down to 5 dimensions, two way to obtain a BPS state with charge vector $k\mathbf{q}$ in 5 dimensions: we could wrap the \mathbf{q} -string k times around the circle, or we could wrap the $k\mathbf{q}$ once. But, in the heterotic theory, there is only one BPS state with charge vector $k\mathbf{q}$. So, since the k -times wrapped \mathbf{q} -string *must* exist, the $k\mathbf{q}$ must not exist as a stable BPS state. Exactly the same argument can be used to rule out bound states at threshold for the (p, q) strings in 10 dimensions [9].

At a generic point in the moduli space, the gauge group in 5 dimensions is $U(1)^{27}$. The

bosonic part of the supergravity Lagrangian is [13]

$$\mathcal{L} = (M_{pl}^{(5)})^3 \sqrt{-g} \left[-\frac{1}{2} R - \frac{1}{4(M_{pl}^{(5)})^2} e^{2\varphi_H/3} a_{ij} F^i \cdot F^j - \frac{1}{4(M_{pl}^{(5)})^2} e^{-4\varphi_H/3} F^2 \right. \\ \left. - \frac{1}{6} (\partial\varphi_H)^2 - \frac{1}{2} \gamma_{\alpha\beta} \partial\phi^\alpha \partial\phi^\beta \right] + \frac{\sqrt{2}}{8} C_{ij} F^i \wedge F^j \wedge A \quad . \quad (3.8)$$

Here, as before, the ϕ parametrize the moduli space, \mathcal{M} , and $\gamma_{\alpha\beta}$ is the $SO(21, 5)$ -invariant metric. φ_H is a scalar in the gravity multiplet, which is either the heterotic string dilaton or, using (3.3), the dilatation mode of the circle in the IIB picture. A is the graviphoton (with field strength $F = dA$), A^i (with field strengths $F^i = dA^i$) are 26 $U(1)$ gauge fields, 21 of which are part of 5-D vector multiplets, and 5 of which transform as the $\mathbf{5}$ of $Sp(2)$ and are part of the gravity multiplet. C_{ij} is the constant matrix

$$C_{ij} = E_i(\phi) \cdot E_j(\phi) = e_i \cdot e_j \quad (3.9)$$

and

$$a_{ij} = 2E_i(\phi)_+ \cdot E_j(\phi)_+ - C_{ij} \quad . \quad (3.10)$$

In the heterotic string picture, the $U(1)^{26}$ is simply the unbroken gauge group that one has at a generic point in the Narain moduli space. The 27th $U(1)$, the graviphoton, arises from the antisymmetric tensor field, $B_{\mu\nu}$, which is dual to a 1-form in 5 dimensions. Among the $U(1)^{26}$ gauge bosons, 5 are “right-moving”, while the rest are “left-moving”. The “right-moving” gauge bosons transform in the $\mathbf{5}$ of $Sp(2)$, and are part of the gravity supermultiplet.

In the IIB picture, the graviphoton arises as the Kaluza-Klein reduction of the 6-dimensional metric, compactified on S^1 . The other 26 gauge fields arise as the reductions of the 2-forms that were present in 6 dimensions. (Since we decomposed the 2-forms in 6 dimensions into their self- and anti-self-dual pieces we can, without loss of generality, take one of the indices of each to be tangent to the S^1 .) Again, 5 correspond to self-dual and 21 to anti-self-dual 2-forms.

At special points in the moduli space, the $U(1)^{21}$ symmetry (excluding the graviphoton, and the 5 “right-moving” photons in the gravity multiplet) gets enhanced to a nonabelian gauge symmetry. In the heterotic picture, this occurs when there are vectors in the Narain lattice with $\mathbf{q}_R = 0$, $q_L^2 = 2$. But, given the correspondence (3.2), we see that this is precisely where the IIB theory develops tensionless strings (2.15). The cartan generators correspond, as we have seen, to anti-self-dual 2-forms in 6 dimensions. The rest of the generators correspond to tensionless strings, wrapped around the S^1 .

Using (3.3),(2.14), we see that the inverse gauge couplings,

$$\left(\frac{1}{g^2}\right)_{ij} = M_{pl}^{(5)} e^{2\varphi_H/3} a_{ij} = M_s a_{ij}$$

are given by essentially the same expression as we found above, (3.1),(2.10), for the masses of the wrapped strings. The graviphoton inverse gauge coupling is

$$\frac{1}{g^2} = M_{pl}^{(5)} e^{-4\varphi_H/3} = \frac{2^{3/4}}{M_s^{1/2} r^{3/2}}.$$

which becomes weakly-coupled as we take the radius $r \rightarrow 0$.

The gauge coupling near the A_1 singularity discussed in section 2.1 is

$$\left(\frac{1}{g^2}\right)_{ij} = \frac{1}{\sqrt{2}} (M_{pl}^{(5)})^2 r \begin{pmatrix} e^{2\varphi} & 0 \\ 0 & e^{-2\varphi} \end{pmatrix}$$

and so, near the singularity we have two strongly coupled fields which degenerate while away from the singular point one becomes weakly coupled and the other strongly coupled. Note that near the singularity we have a Z_2 symmetry which exchanges the two entries in the matrix. This is a typical remnant of an A_1 singularity. Up on the covering space, $SO(21, 5)/SO(21) \times SO(5)$, the gauge couplings are smooth near the phase transition points associated to tensionless strings. However, when we mod out by the discrete symmetry $SO(21, 5, \mathbb{Z})$, these transition points are orbifold singularities in \mathcal{M} .

One naïvely might wonder about the tensionless strings which are *not wrapped* around the S^1 , but which propagate in the 5-dimensional spacetime. Since, in 5 dimensions, a string is dual to a particle, these strings are *magnetic* sources for the gauge fields we have been studying. For small r , they are very heavy compared to the typical particle mass (3.1). So, even in the limit where both are going to zero (that is, when the underlying string in 6 dimensions is becoming tensionless), the particle, being so much lighter, dominates the infrared physics.

3.1. Strong heterotic coupling

One of the most remarkable developments in this field was the observation by Witten [14] and Townsend [15] that the strong coupling limit of type IIA string theory in 10 dimensions is a theory with 11-dimensional Lorentz invariance. Here we see that, as a consequence of (3.3), the strong coupling limit of the heterotic string in 5 dimensions (compactified on T^5) is a theory with 6-dimensional Lorentz-invariance, the type IIB string compactified on $K3$!

Of course, in this limit of large heterotic coupling, or large r , the situation described in the last paragraph of the previous subsection is turned on its head. In this case, it is the

magnetically-charged *strings* which are much lighter than the electrically-charged *particles*. It is the *strings* which dominate the infrared physics¹.

The BPS saturated particle multiplets of theories with enough supersymmetry can be classified into *short* (annihilated by half of the supersymmetries) and *ultrashort* (annihilated by 3/4 of the supersymmetries) multiplets. In the case of toroidally-compactified heterotic strings, these correspond to

short $N_R = 0$

ultrashort $N_L = N_R = 0$

We saw that those BPS strings which can become tensionless always have $q^2 = -2$, *i.e.*, they correspond to ultrashort multiplets upon compactification. The strings (like the fundamental string) with $q^2 \geq 0$ which never become tensionless correspond to short multiplets. The concept of short and long multiplets certainly makes sense for the dimensionally-reduced supersymmetry algebra (3.6). Perhaps a similar concept can be made sense of for *strings* in the 6 dimensional (0,2) supersymmetry algebra (2.1).

4. M-Theory Picture

It was shown in [16,17] that the dual theory to type IIB on $K3$ is M-theory on a T^5/\mathbb{Z}_2 . The \mathbb{Z}_2 acts as -1 on all circles of the torus. It also acts on the 3-form gauge field as -1 . This symmetry breaks half of the supersymmetries and leaves all generators which obey $\Gamma_{7,8,9,10,11}\epsilon = \epsilon$. This theory gives chiral supersymmetry in six dimensions, namely the (0,2) supersymmetry discussed above. The condition on the generators is consistent with having 5-branes which are localized in the 7, 8, 9, 10, 11 directions. Actually an anomaly cancellation argument leads to having 16 five branes. It was stressed in [17] that gravitational anomalies must cancel locally in spacetime. This led to the observation that the 32 fixed points on the torus carry magnetic charge $-\frac{1}{2}$ (in the units where each of the 16 five-branes carries magnetic charge $+1$). In addition to cancelling the anomalies, this satisfies magnetic charge conservation.

The untwisted sector contains 5 2-form fields, given by integrating the 3-form of M-theory over a 1-cycle in T^5 . Since both the 3-form and the 1-cycle are odd under the \mathbb{Z}_2 , these states are even, and survive the orbifold projection. The self-dual parts of these 2-forms correspond to the 5 self-dual 2-forms in the (0,2) gravity multiplet. The anti-self-dual parts give rise

¹Of course, our ability to make these statements, in the limit where (3.8) becomes strongly-coupled, relies on the fact that the N=4 supersymmetry forbids corrections to the BPS formulæ for these masses.

to 5 tensor multiplets. In addition we have 10 scalars from the 3-form field (with all indices tangent to T^5) and 15 scalars from the modes of the metric on T^5 . Together with the 5 anti-self-dual 2-form fields they form the 5 tensor multiplets. These are the massless fields in the untwisted sector.

In addition to the tensor multiplets from the untwisted sector, there are 16 more tensor multiplets, one carried by each of the 5-branes. Each tensor multiplet has an anti-self-dual 2-form and five scalars, which give the position of the 5-brane on T^5/\mathbb{Z}_2 .

The M-theory description of the 105-dimensional moduli space clearly has 25 untwisted moduli related to the geometry of the T^5 and 80 twisted moduli related to the positions of the 5-branes. It is tempting to try to identify the former with the moduli of the torus in the compactification of the heterotic string on T^5 and the latter with the Wilson lines of the heterotic compactification. This is almost, but not quite, correct.

Let X be the T^5 of the M-theory compactification. The space of twisted moduli is

$$X^{16}/\Gamma \tag{4.1}$$

where Γ is the group $S_{16} \ltimes (\mathbb{Z}_2)^{16}$ with generators

$$\begin{aligned} \sigma_{ij} : \vec{x}_i &\leftrightarrow \vec{x}_j \\ s_i : \vec{x}_i &\rightarrow -\vec{x}_i \quad . \end{aligned} \tag{4.2}$$

The reason for modding out by Γ is evident. Since we do not label the 5-branes, we should mod out by transformations which permute them. Also, since the 5-branes are really propagating on X/\mathbb{Z}_2 , not X , we should consider \vec{x}_i and $-\vec{x}_i$ as equivalent.

Let us compare this with the space of Wilson lines of the $Spin(32)/\mathbb{Z}_2$ heterotic string on $Y = T^5 = \mathbb{R}^5/\Lambda$. This is

$$\text{Hom}(H_1(Y), u(1)^{16})/\text{identifications} = (Y^*)^{16}/\Gamma \tag{4.3}$$

where $Y^* = \mathbb{R}^5/\Lambda^*$ is the dual torus to Y , obtained by modding out \mathbb{R}^5 by the lattice dual to Λ . Here we note that Γ is the group of automorphisms of the root lattice of $SO(32)$ (which is also the group of automorphisms of the $Spin(32)/\mathbb{Z}_2$ weight lattice). This agrees with (4.1), provided we identify the M-theory torus X , not with the heterotic torus Y , but with its dual torus Y^* .

The fact the X is to be identified with Y^* is crucial, as well, to understanding how the untwisted moduli of the two theories map onto each other. In trading the torus for the dual torus, we naturally exchange the 3-form $C \in H^3(X)$ of M-theory with the 2-form $B \in H^2(Y)$ of the heterotic string. Modulo a slight subtlety, which we will encounter in section 4.2,

to do with relating the overall volume of X to that of Y , we now understand the mapping between the heterotic and M-theory moduli spaces.

With this description of the M-theory moduli space in hand, we can examine the M-theory description of enhanced symmetry groups (which, upon compactification down to 5 dimensions, become gauge groups) arising in the limit as some strings become tensionless.

4.1. The twisted sector

Consider n 5-branes in the bulk. There is a $U(1)^n$ symmetry, carried by the ASD 2-forms in the n tensor multiplets carried by the n 5-branes. Let these n 5-branes approach each other in the bulk. We need to tune $5(n-1)$ real parameters to do this. We develop an $U(n) \sim SU(n) \times U(1)$ symmetry. The $U(1)$ is the “center of mass” tensor multiplet. Open 2-branes which stretch between pairs of 5-branes give rise to strings which become tensionless in this limit. These strings are charged with respect to the tensor multiplets carried by the respective 5-branes. The 2-brane which stretches between the i^{th} and j^{th} 5-brane has charge $(0, \dots, 1_i, 0, \dots, -1_j, 0, \dots, 0)$, or minus this, depending on its orientation. The corresponding strings are thus in 1-1 correspondence with the roots of $U(n)$. This point, where n 5-branes coincide is an S_n orbifold point in the moduli space \mathcal{M} , as we see from (4.1),(4.2). This is to be expected, as S_n is the Weyl group of the A_{n-1} root lattice.

Now let this collection of n 5-branes approach one of the fixed points. This requires tuning 5 more real parameters. In the limit, $U(n)$ is promoted to $SO(2n)$. The new tensionless strings come from 2-branes which stretch between two five branes, passing through the fixed point on the way. Since the resulting strings effectively change orientation when they pass through the fixed point, these strings have charge vectors of the form $(0, \dots, 1_i, 0, \dots, 1_j, 0, \dots, 0)$, or minus this. Together with the previous strings, these form the roots of $SO(2n)$. This point in the moduli space is fixed not just by permutations of the positions of the n 5-branes, but also by reflections of those positions through the origin. Thus the symmetry group is $S_n \ltimes (\mathbb{Z}_2)^n$, the automorphism group of D_n ².

No surprise, the groups which arise by allowing the 5-branes to move about in this way are exactly those that arise when tuning the Wilson lines in the $Spin(32)/\mathbb{Z}_2$ heterotic string.

4.2. The untwisted sector

To see other enhanced symmetry groups, we need to tune also the moduli in the untwisted sector. The simplest thing we would like to see is the $SU(2)$ symmetry which arises when

²For $n \neq 4$. The full automorphism group of D_4 is $S_3 \ltimes (S_4 \ltimes (\mathbb{Z}_2)^3)$, but only part of this is a subgroup of Γ .

we take one of the radii of the torus Y to the self-dual radius³.

One subtlety, which we now encounter, is that the mapping $(\text{vol}(Y), e^{\varphi_H}) \rightarrow (\text{vol}(X), r)$ mixes these variables in a nontrivial way. This is easily seen, for instance, from (3.3). The $M_{pl}^{(5)}$ that appears there differs from the 11-dimensional M_{pl} by a factor proportional to $(r \text{vol}(X))^{1/3}$.

We can realize the T-duality of the heterotic theory on Y as a symmetry of the 6-dimensional M-theory compactification (that is, without transforming r) *provided* we make compensating changes in the other radii so as to leave the overall volume fixed. In the case at hand, the relevant choice is to fix the volume of X to be

$$\text{vol}(X) = \left(\frac{2\pi}{M_{pl}} \right)^5. \quad (4.4)$$

The strings that arise in the untwisted sector consist of 5-branes wrapped around 4-cycles (which we will call **A**-strings) on X and membranes wrapped around 1-cycles⁴ (which we will call **B**-strings). The name, **B**-string, is apt. Integrating the 3-form around *one of* the 1-cycles on X , we obtain a 2-form, which is to be identified with the B field of the type IIB string in 6 dimensions. The **B**-string associated to this 1-cycle couples to this 2-form and *is* the fundamental IIB string.

Consider the **B**-string associated to the 1-cycle, γ , on X . It coupled to a certain 2-form in 6 dimensions, which we might call $B = (B^- + B^+)$. As in section 2.1, a candidate tensionless string arises as the bound state of this **B**-string with a string which couples to the dual 2-form $(B^- - B^+)$. The 5-brane couples to the dual of the 3-form, so the natural candidate for the string we are after is an **A**-string wrapped around the 4-cycle γ^* dual to γ .

With vanishing moduli of the 3-form C , the tension of the **B**-string formed by wrapping the membrane around a circle of radius R is

$$2\pi R T^{(2)} = \frac{M_{pl}^3 R}{4\pi} \quad (4.5)$$

and the tension of the **A**-string wrapped around the dual T^4 is

$$\text{vol}(\gamma^*) T^{(5)} = \frac{(2\pi)^4}{M_{pl}^5 R} T^{(5)} = \frac{M_{pl}}{8\pi R} \quad (4.6)$$

where $T^{(2)}$ and $T^{(5)}$ are the membrane and 5-brane tensions.

³This is $R^2 = \alpha' = 2M_s^{-2}$ in the *string metric*. In the 11-dimensional *Einstein* metric, it is $R^2 = \frac{1}{2}M_{pl}^{-2}$, roughly because X is the dual torus to Y .

⁴A 1-cycle is odd under the \mathbb{Z}_2 symmetry, but so is the 3-form gauge field which couples to the membrane. So these membranes survive the orbifold projection.

When the radius of the circle, γ , is $\frac{1}{\sqrt{2}}M_{pl}^{-1}$, the tensions of the **A**-string and the **B**-string are equal and of $\mathcal{O}(M_{pl}^2)$, but their bound state becomes tensionless. This is the string which we studied in section 2.1⁵. In the heterotic picture, the **B**-string is a mode of the heterotic string with momentum around the cycle corresponding to γ . The **A**-string is a heterotic string which *winds* around the same cycle. T-duality exchanges momenta and windings, and so exchanges these two strings. Hence we see that the T-duality of the heterotic string is an “electric-magnetic” duality of M-theory, which exchanges the membrane with the 5-brane!

Aside from things like the BPS tension (which is unaffected by quantum corrections), it is hard to study these strings directly, given our current rudimentary knowledge of M-theory. The relevant radii of the torus are $\mathcal{O}(1)$, and we are far from the regime where low-energy 11-dimensional supergravity is valid. Nevertheless, it is important to pursue the matter, as we wish to learn whatever we can about the behaviour of M-theory beyond the realm of validity of the 11-dimensional supergravity approximation.

For instance, by further tuning the locations of the 5-branes from the twisted sector, we obtain further enhancements of the symmetry group (up to $SO(34)$ in this case). The locations in the moduli space where these and other enhanced symmetry groups (*e.g.*, $E_8 \times E_8$) occur are all well-understood on the heterotic side. We hope to provide a fuller account of what these look like in the M-theory picture in a future work.

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⁵To make contact with (2.13), note that $(M_{pl}^{(6)})^4 = \frac{1}{2} \left(\text{vol}(X) \left(\frac{M_{pl}}{2\pi} \right)^5 \right) M_{pl}^4$, where the $\frac{1}{2}$ is due to the fact that the volume of the orbifold is half the volume of X .

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